Math 302 Worksheet 1: Divisibility

Math 302: Introduction to Proofs via Number Theory

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Definition. The set of *natural numbers* is $\mathbb{N} = \{1, 2, 3, 4, ...\}$.

Definition. The set of *integers* is $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

Definition. Let a and b be integers. Then a *divides* b (which we denote by $a \mid b$) if there is an integer k such that $ak = b$. In this case, we call a a *divisor* of b, and b a *multiple* of a.

Problem 1.1. List some values of x such that $x \mid 12$. Now list some values of y such that $12 \mid y$.

Problem 1.2. Explain the difference between $a \mid b$ and $\frac{a}{b}$.

Problem 1.3. Suppose that a, b and c are integers and that $a \mid b$ and $a \mid c$. How is a related to $b + c$? To $b - c$? To bc? Try some examples to test your theories.

Problem 1.4. Let a and b be nonzero integers. What can you conclude if $a \mid b$ and $b \mid a$?

Problem 1.5. Are there integers a, b and c such that $a \mid bc$ but $a \nmid b$ and $a \nmid c$? Can you find an integer a such that for all integers b and c, if $a \mid bc$ then either $a \mid b$ or $a \mid c$?

Given two integers a and b with $b > 0$, it is not necessarily true that $a \mid b$ or that $b \mid a$. However, a can always be described in terms of its distance from a multiple of b. For example, 5 does not divide 32 (we can write $5 \nmid 32$), but $5 \mid 30$, and $32 = 30 + 2 = 5 \times 6 + 2$.

Problem 1.6. Using the example above as a model, write equations relating:

(a) 27 and a multiple of 4

(b) 40 and a multiple of 17

(c) -15 and a multiple of 6

Are there different ways to do this?

Problem 1.7. For general integers a and b, express a in terms of its distance from a multiple of b. Is your expression unique? How can it be made unique?